Chapter 2: Feedback Theory

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Contents

1 Negative Feedback Systems

Consider what we have learnt in Control Systems. Figure 1 shows the negative feedback system that most of us are familiar with, while equation 1 describes the input-output relationship of such a system.

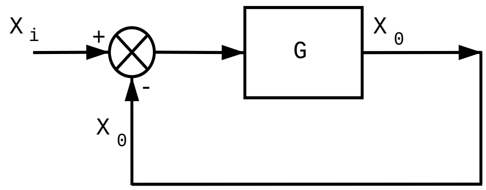

Figure 1: An Example of a Negative Feedback System

$$G \cdot (X_i - X_0) = X_0$$
$$\frac{X_0}{X_i} = \frac{G}{1+G} \approx 1 \qquad (1)$$

If the *loop gain*, G, is much greater than one, then the output, X_0, follows the input, X_i, and the error, X_i - X_0, tends to zero.

Alternately, the input arrangement could be such that we have a summing device at the input instead of the error detector. The block diagram and the resulting input-output relationship are shown in figure 2 and equation 2, respectively.

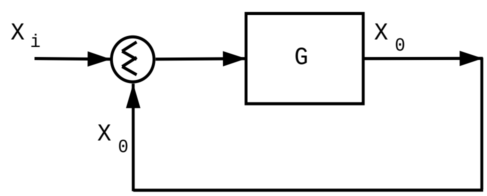

Figure 2: An Alternate Feedback System

$$\frac{X_0}{X_i} = \frac{G}{1-G} = \frac{-1}{1-\frac{1}{G}} \qquad (2)$$

Here, G is known as the loop gain. If it is -G_0 where G_0 is a positive number, the situation is termed as *negative feedback*. If G = G_0. the system is said to have *positive feedback*. A negative feedback system is described by equation 3 while a positive feedback system is described by equation 4.

$$\text{For negative feedback, } G_f = \frac{-1}{1 + \frac{1}{G_0}} \tag{3}$$

$$\text{For positive feedback, } G_f = \frac{-1}{1 - \frac{1}{G_0}} \tag{4}$$

2 Features of Feedback

2.1 Desensitization of the System Response to Active Parameter Variations

Negative feedback systems are designed such that the steady state error goes to zero, i.e., for a finite output, X_0, the output of the summing device is given by X_i + X_0, and is equal to $\frac{X_0}{G_0}$. As G_0 tends to infinity, the output of the summing device goes to zero. In other words, X_i + X_0 is zero. Hence, X_0 = - X_i, a relation independent of the gain, G_0 (We must remember, though, that this is true only if the gain, G, is infinity). We can hence conclude that negative feedback desensitizes the system response to the variations in the active parameter, G.

We just proved that the system response is insensitive to the active parameter if its gain is infinity. We now need to quantify the sensitivity of the system response to the variations in G, when G is finite. To answer this question, let us first try to understand how sensitivity is defined.

Definition of Sensitivity

$$\text{Sensitivity Function, } {S_X}^Y = \frac{\frac{\partial Y}{Y}}{\frac{\partial X}{X}} = \frac{\partial Y}{\partial X} \cdot \frac{X}{Y}$$

To redefine the above equation in words, the *sensitivity of Y to X* is defined as the percentage variation of Y for a percentage variation of X.

Sensitivity is an important parameter in electronics for assessing the performance of various systems. When an electronic system is designed, we have to understand the sensitivity of the performance factors of the system to active and passive device paramters used in building the system.

As an example, consider the LC oscillator shown in figure 3. The losses of the inductor, L, and the capacitor, C, can be represented by the resistor, R_p. These losses are compensated by simulating a negative resistance, $R_p^{'}$, across the parallel RLC circuit.

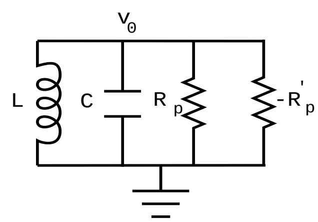

Figure 3: LC Oscillator Circuit

The circuit behaviour is governed by the set of differential equations described in equations 5 - 6.

$$\Sigma i = 0$$

$$\frac{1}{L}\int v_0\,dt + C\frac{dv_0}{dt} + \frac{v_0}{R_p} - \frac{v_0}{R_p^{'}} = 0 \tag{5}$$

$$\frac{d^2 v_0}{dt^2} + \frac{1}{CR_p}\cdot(1 - \frac{R_p}{R_p^{'}})\cdot\frac{dv_0}{dt} + \frac{v_0}{LC} = 0 \tag{6}$$

When $R_p^{'} = R_p$ is sustains a sinusiodal oscillation

$$v_0 = V_{pm} sin(\frac{t}{\sqrt{LC}} + \phi_0$$

If $R_p^{'} > R_p$ the oscillations *die down*

If $R_p^{'} < R_p$ the oscillations *grow*

$$\textit{Frequency of Oscillation}, \omega_0 = \frac{1}{\sqrt{LC}} \tag{7}$$

From the above set of equations, it is clear that the frequency of oscillation, ω_0, is determined only be the values of inductance and capacitance.

In the above example, an active device is used to simulate the negative resistance, $R_p^{'}$. Since the frequency of oscillation is independent of $R_p^{'}$, we can conclude that the sensitivity of the frequency of oscillation to R_p is zero, i.e., $S_{R_p'}^{\omega_0} = 0$. The sensitivity of the frequency of oscillation to L or C is $-\frac{1}{2}$ (How?).

Now let us return to the discussion on the negative feedback system. Consider the system response given by equation 8. It can be shown that the sensitivity of the system response to the active parameter depends on the loop gain, and is given by equation 9.

$$G_f = \frac{-1}{1 + \frac{1}{G_0}} \tag{8}$$

$$S_{G_0}^{G_f} = \frac{-1}{1 + G_0} \tag{9}$$

From the above equations, it is clear that the sensitivity to the active parameter has been scaled down by a factor of $(1 + G_0)$ (It would be worthwhile to note that the gain also scaled down by the same parameter). This is one of the most important benefits of negative feedback.

Let us now consider positive feedback in more detail. The dependence of the gain, G_f, on the active parameter, G_0, is given in equation 10.

$$G_f = \frac{-1}{1 - \frac{1}{G_0}} \tag{10}$$

When $G_0 = 1$, the output becomes infinite irrespective of the input and the system becomes *unstable*. To obtain a finite output, we must maintain G_0 below unity. In this region, the system is stable, but very sensitive to variations in G_0. The sensitivity increases as G_0 approaches unity. Hence, in amplifier design and control system design, only negative feedback with $G_0 \gg 1$ is used.

2.2 Noise and Distortion Reduction

We will now discuss the benefits of negative feedback in the reduction of noise and distortion. Consider the block diagram shown in figure 4.

The system transfer function for such a system is derived in equations 11 - 14.

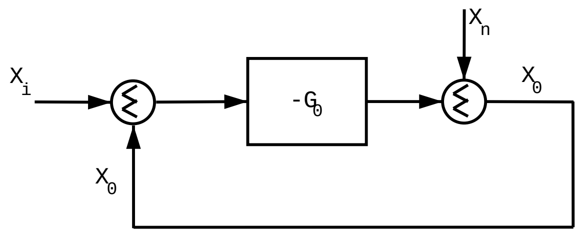

Figure 4: Influence of Output Disturbances in a Negative Feedback System

$$\begin{aligned} X_0 &= -(X_i + X_0) \cdot G_0 + X_n & (11) \\ X_0 \cdot (1 + G_0) &= -X_i \cdot G_0 + X_n & (12) \\ X_0 &= \frac{-1}{1 + \frac{1}{G_0}} \cdot X_i + \frac{\frac{X_n}{G_0}}{1 + \frac{1}{G_0}} & (13) \\ &= -X_i + \frac{X_n}{G_0} \qquad \text{if } G_0 >> 1 & (14) \end{aligned}$$

From these equations, we can conclude that noise and distortion (as long as they are within the loop) are scaled down by the loop gain.

2.3 Effect of Finite Bandwidth of the Active Device used in Negative Feedback Systems

The finite bandwidth of the active device can be modeled as shown in equation 15. In the equation, ω_d represents the bandwidth of the gain stage, and G_0 denotes the dc gain.

$$G = \frac{-G_0}{1 + \frac{s}{\omega_d}} \qquad (15)$$

The system transfer function can now be derived in terms of both G_0 and ω_d and is shown in equations 16 - 18.

$$G_f = \frac{-1}{1+\frac{1}{G}} \tag{16}$$

$$= \frac{-1}{1+\frac{1}{G_0}+\frac{s}{G_0\cdot\omega_d}} \tag{17}$$

$$\approx \frac{-1}{1+\frac{s}{G_0\cdot\omega_d}} \quad \text{if } G_0 >> 1 \tag{18}$$

The *bandwidth* of the feedback system response $= G_0 \cdot \omega_d =$ (dc loop gain)(bandwidth of the loop) = gain-bandwidth product.

3 Applications of Negative Feedback

Let us consider the feedback system shown in figure 2. The first modification that we can do to the negative feedback system shown in figure 2 is to introduce some signal-processing activity in the feedback path. This signal-processing block could be linear or non-linear as the application may demand. This is described in figures 5 and 6. The transfer function of the linear system is derived in equations 19 - 23 while the transfer function (and the requisite conditions) for the non-linear system are detailed in equations 24 - 26.

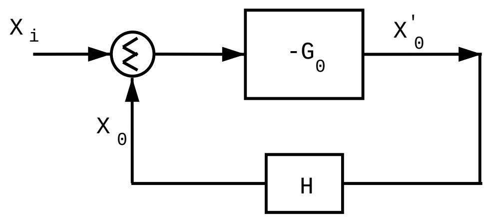

Figure 5: Linear System that Obtains Reciprocal of the Input

$$\text{loop gain } = G_0 \cdot H \gg 1 \tag{19}$$

$$\text{if loop gain is large, } X_i + X_0 = 0 \tag{20}$$

$$X_0 = -X_i \tag{21}$$

$$H \cdot X_0' = -X_i \tag{22}$$

$$X_0' = -\frac{X_i}{H} \tag{23}$$

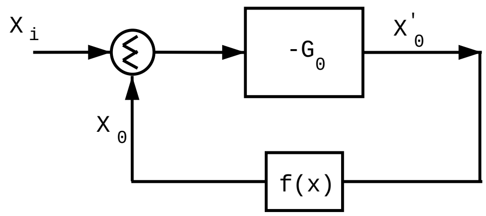

Figure 6: Non-Linear System that Provides Inverse Function of the Input

$$X_0 = f(X_0^{'}) \ , \tag{24}$$

a non-linear system such that $\frac{\partial X_0}{\partial X_0^i}$ is always positive and $G_0 \cdot \frac{\partial X_0}{\partial X_0^i} \gg 1$

$$X_i = -X_0 \tag{25}$$

$$X_0^{'} = f^{-1}(-X_i) \tag{26}$$

Figures 8 to 18 describe various systems that use negative feedback. It will be well worth noting that although these systems perform different functions and are used in completely different applications, the underlying principles are the same. In fact, a number of considerations, e.g. stability, are similar in all these systems, and hence, we will discuss some of these common issues later in this chapter.

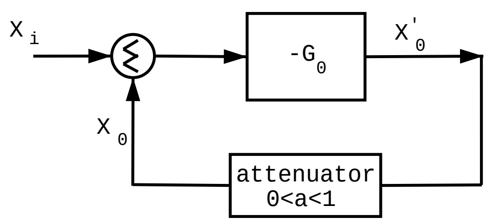

Figure 7: Amplifier Obtained Using an Attenuator in Feedback; $X_0^{\prime} = -\frac{X_i}{a}$

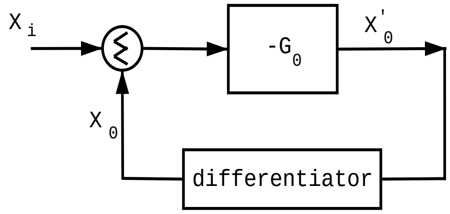

Figure 8: Integrator Obtained Using a Differentiator in Feedback; $X_0' = -\int X_i \, dt$

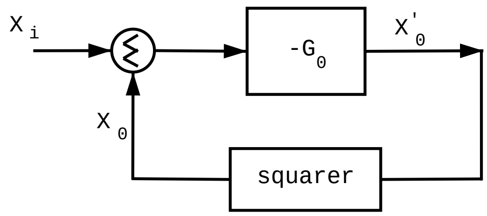

Figure 9: Square-root Function Obtained by Using a Squarer in Feedback; $X_0' = \sqrt{-X_i}$

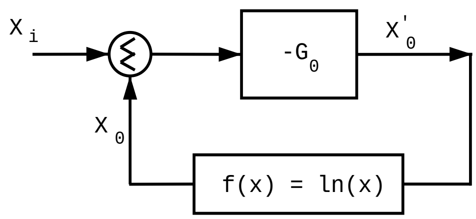

Figure 10: Anti-Log Function Obtained by Using a Log Function in Feedback; $X_0' = e^{-X_i}$

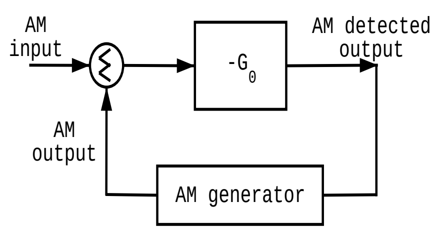

Figure 11: AM Detection Circuit Using an AM generator in Feedback

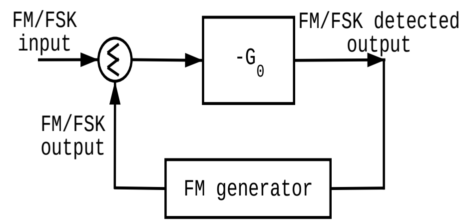

Figure 12: FM Detection Circuit Using an FM generator in Feedback

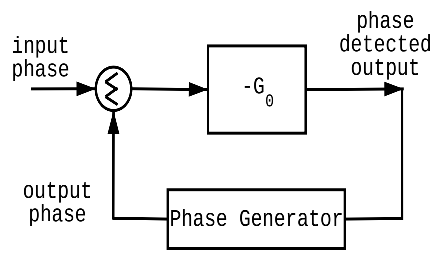

Figure 13: Phase Locked Loop Using a Phase Generator in Feedback

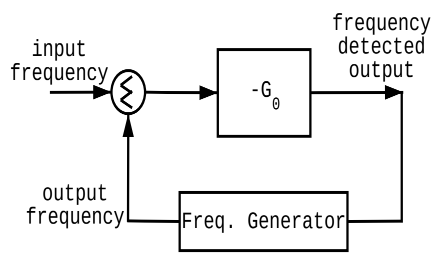

Figure 14: Frequency Locked Loop Using a Frequency Generator in Feedback

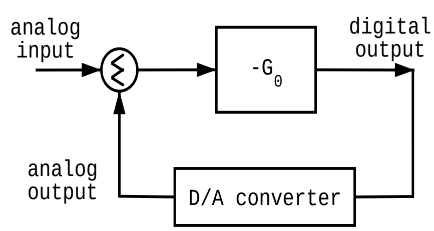

Figure 15: $\Delta - \Sigma$ ADC Using a DAC in Feedback

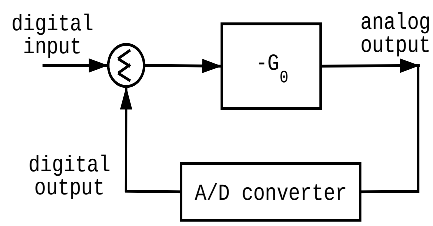

Figure 16: $\Delta - \Sigma$ DAC Using a ADC in Feedback

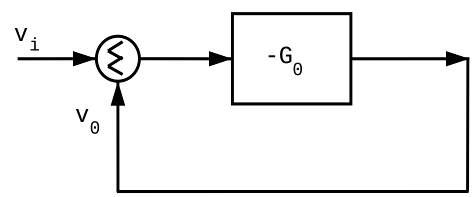

Figure 17: Voltage Follower - Unity Gain Feedback

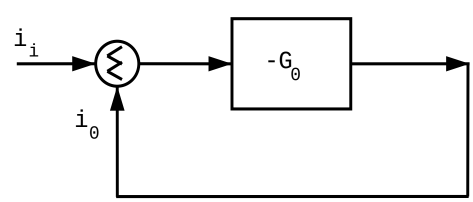

Figure 18: Current Follower - Unity Gain Feedback

4 Stability and Frequency Compensation in Negative Feedback Systems

4.1 Single-pole Roll-off for the Loop Transfer Function

In the previous section, we considered the effect of a single pole in the loop gain of a negative feedback system. The behaviour of such a system, as we saw earlier, can be described using equations 27 - 28

$$G_f = -\frac{1}{1-\frac{1}{G}} \text{ with } G = -\frac{G_0}{1+\frac{s}{\omega_d}}\ , \tag{27}$$

where G_0 is the magnitude of the loop gain and ω_d is the bandwidth of the loop gain

$$G_f = -\frac{1}{1+\frac{1}{G_0}+\frac{s}{G_0 \cdot \omega_d}} \approx \frac{1}{1+\frac{s}{G_0 \cdot \omega_d}} \text{ for } G_0 \gg 1 \tag{28}$$

i.e. the system response has a bandwidth given by (loop bandwidth)(loop dc gain)

In terms of transient response, it means that if $G_0 \gg 1$, the steady state error approaches zero and the system responds to a step input with a time constant given by $\tau = \frac{1}{G_0 \cdot \omega_d}$.

Figure 19 shows one such response. In this case, the time-constant is assumed to be 0.5.

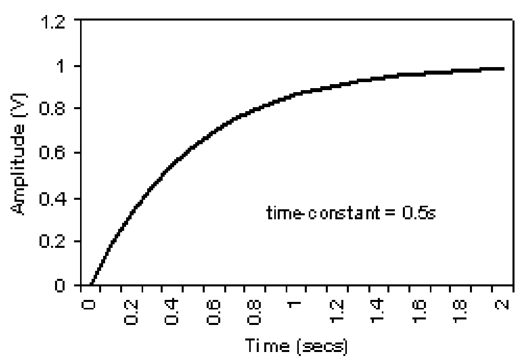

Figure 19: Step-Response of a First-Order System

Figure 20 shows the magnitude and phase response of such a system.

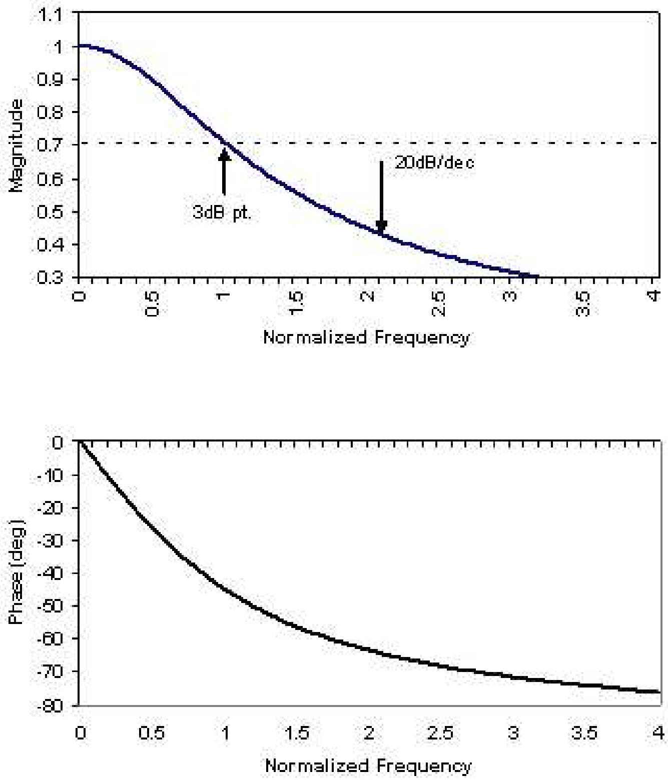

Figure 20: Frequency Response of a First-Order System

Let us now assume that a zero is introduced in the loop gain as shown in equation 29,

$$G = -G_0 \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_d}} \tag{29}$$

This causes the system to permit high-frequency noise to come through and the Signal-to-Noise Ratio (SNR) of the system deteriorates. Hence, this kind of compensation will not suffice. The loop transfer function should be such that the order of the numerator polynomial is at least one order less than that of the denominator polynomial for limiting the noise in the system. (i.e. the system has to be band-limited).

Based on this requirement of band-limiting the signal, we can derive a condition for the location of the zero (Equations 30 - 35).

$$\begin{aligned} G_f &= -\frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_z} + \frac{1}{G_0} + \frac{s}{G_0 \cdot \omega_d}} && (30) \\ &\approx -\frac{1 + \frac{s}{\omega_z}}{(1 + \frac{s}{\omega_z}) + \frac{s}{G_0 \cdot \omega_d}} && (31) \\ & && (32) \end{aligned}$$

For the signal to be band-limited,

$$\begin{aligned} \frac{\frac{1}{\omega_z}}{\frac{1}{\omega_z} + \frac{1}{G_0 \cdot \omega_d}} &\ll 1 && (33) \\ \frac{1}{1 + \frac{\omega_z}{G_0 \cdot \omega_d}} &\ll 1 && (34) \\ \frac{\omega_z}{G_0 \cdot \omega_d} &\gg 1 && (35) \end{aligned}$$

4.2 Double-pole Roll-off for the Loop Gain

Assuming the transfer function described in equation 36 for the active device, equations 37 - 40 compute the overall transfer function for the negative feedback system.

$$
\begin{aligned}
G &= -\frac{G_0}{(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d1}})} && (36) \\
G_f &= -\frac{1}{1+\frac{1}{G_0}\cdot(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d1}})} && (37) \\
&= -\frac{1}{1+\frac{1}{G_0}+\frac{s}{G_0}\cdot(\frac{1}{\omega_{d1}}+\frac{1}{\omega_{d2}})+\frac{s^2}{\omega_{d1}\omega_{d2}G_0}} && (38) \\
&= -\frac{1}{1+s(\frac{1}{G_0\omega_{d1}}+\frac{1}{G_0\omega_{d2}})+\frac{s^2}{G_0\omega_{d1}\omega_{d2}}} && (39) \\
&= -\frac{1}{1+\frac{s}{\omega_0 Q_0}+\frac{s^2}{\omega_2^2}} && (40)
\end{aligned}
$$

ω_0 is known as the natural frequency of the system and Q_0 is defined as the quality factor (also represented as $\frac{1}{2\xi}$ where ξ is called the damping factor). The denominator polynomial of equation 40 can now be solved to obtain the poles of the system as shown in equation 41

$$\frac{s}{\omega_0} = -\frac{1}{2\cdot Q_0} \pm \sqrt{\frac{1}{4\cdot Q^2}-1} \qquad (41)$$

For $Q_0 < \frac{1}{2}$, the roots lie on the negative real axis on the s-plane
For $Q_0 = \frac{1}{2}$, the roots are coincident on the negative real axis on the s-plane
For $Q_0 > \frac{1}{2}$, the roots are complex and are given by,

$$\frac{s}{\omega_0} = -\frac{1}{2\cdot Q_0} \pm j\sqrt{1-\frac{1}{4\cdot Q^2}}$$

Rederiving the quality factor in terms of G_0, ω_{d1} and ω_{d2}, we have

$$
\begin{aligned}
\omega_0 &= \sqrt{G_0\omega_{d1}\omega_{d2}} && (42) \\
Q_0 &= \frac{\sqrt{G_0}}{\sqrt{\frac{\omega_{d1}}{\omega_{d2}}}+\sqrt{\frac{\omega_{d2}}{\omega_{d1}}}} && (43)
\end{aligned}
$$

If $\omega_{d1} = \omega_{d2}$, Q_0 is maximized

$$Q_{max} = \frac{\sqrt{G_0}}{2} \qquad (44)$$

In order to maintain a low steady-state error, G_0 is required to be high. Consequently, a second-order system will generate high-Q pole pairs. When an impulse is applied to such a system, it will start ringing at its natural frequency. Figures 21 - 22 show the frequency and transient response of such a system with a quality factor of 10.

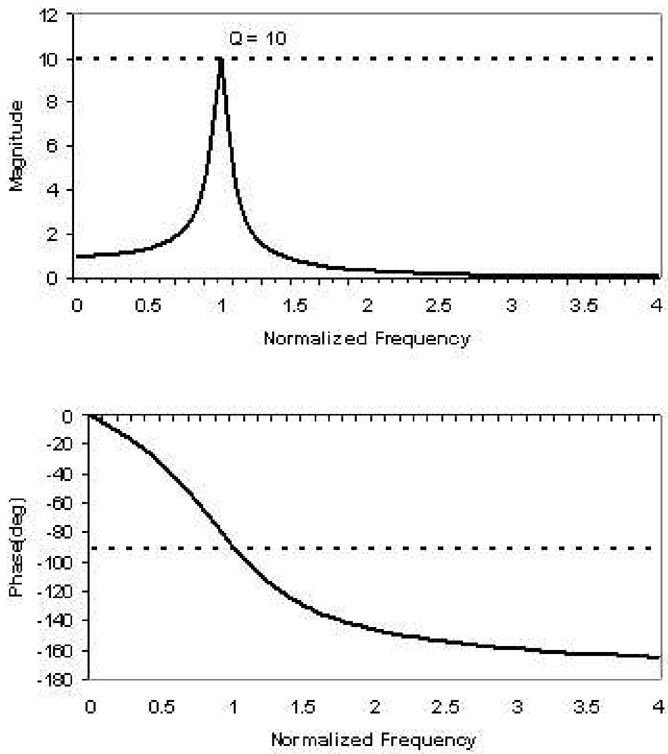

Figure 21: Frequency Response of a Second-Order System; Q = 10

4.3 Summary

A negative feedback system can be a zeroth order, or in other words, a wide-band system. Such a system responds instantaneously to changes at the input. It is a high-speed control system. Such a high-speed control system will respond to noise just as it responds to signal. In order not to degrade the SNR at the output, it is desired that the system be band-limited. From

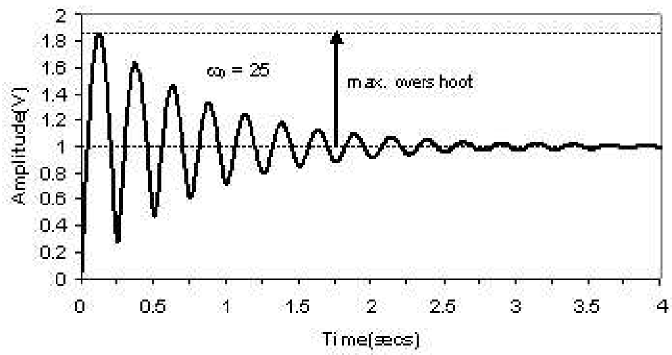

Figure 22: Step-Response of a Second-Order System; Q = 10

such a perspective, the system bandwidth should be chosen to be the same as the signal bandwidth. Such a system will not respond instantaneously to any changes at the input, and its behaviour will be governed by the order. A first-order system will exhibit an exponential response with a time-constant given by the reciprocal of the gain-bandwidth product of the loop. To improve the speed of the response, the order has to be increased to two. The quality factor (Q) of the system can now be manipulated (e.g. $Q = \frac{1}{2}$) to make the system respond fast, and also settle down to its ateady-state value quickly. A high-Q system is not desired as it causes ringing and slow settling.

5 Frequency Compensation of a Second-Order System by Introducing a Zero

Consider an active device, G, whose transfer function is given in equation 45

$$G = \frac{-G_0(1 + \frac{s}{\omega_z})}{(1 + \frac{s}{\omega_{d1}})(1 + \frac{s}{\omega_{d2}})} \tag{45}$$

Then,

$$G_f = \frac{-1}{1 + \frac{(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d2}})}{G_0(1+\frac{s}{\omega_z})}} \tag{46}$$

$$= \frac{-(1+\frac{s}{\omega_z})}{1 + \frac{s}{\omega_z} + \frac{(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d2}})}{G_0}} \tag{47}$$

$$= \frac{-(1+\frac{s}{\omega_z})}{1 + \frac{s}{\omega_z} + \frac{1}{G_0}\cdot(\frac{s}{\omega_{d1}} + \frac{s}{\omega_{d2}}) + \frac{s^2}{G_0\omega_{d1}\omega_{d2}}} \tag{48}$$

Where,

$$\omega_0 = \sqrt{G_0\omega_{d1}\omega_{d2}} \tag{49}$$

$$\frac{1}{\omega_0 Q_0} = \frac{1}{\omega_z} + \frac{1}{G_0\omega_{d1}} + \frac{1}{G_0\omega_{d2}} \tag{50}$$

$$\frac{1}{Q_0} = \frac{\sqrt{G_0\omega_{d1}\omega_{d2}}}{\omega_z} + \sqrt{\frac{\omega_{d1}}{\omega_{d2}G_0}} + \sqrt{\frac{\omega_{d2}}{\omega_{d1}G_0}} \tag{51}$$

$$= \sqrt{G_0}\frac{\sqrt{\omega_{d1}\omega_{d2}}}{\omega_z} + \sqrt{\frac{\omega_{d1}}{\omega_{d2}}}\frac{1}{\sqrt{G_0}} + \sqrt{\frac{\omega_{d2}}{\omega_{d1}}}\frac{1}{\sqrt{G_0}} \tag{52}$$

For a given system with G_0, ω_{d1} and ω_{d2} known, ω_z can be chosen to obtain the required Q_0 depending upon the type of response needed ($Q_0 = \frac{1}{\sqrt{2}}$ for a maximally flat magnitude response, and $Q_0 = \frac{1}{2}$ for a critically damped response).

6 A Third Order System or an Open-Loop Transfer Function with Three Poles

In the preceding section, we considered an active device, G, that had two poles in its transfer function. Let us now consider an active device with three poles in the transfer function.

$$G = \frac{-G_0}{(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d2}})(1+\frac{s}{\omega_{d3}})} \tag{53}$$

Then,

$$G_f = \frac{-1}{1 + \frac{(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d2}})(1+\frac{s}{\omega_{d3}})}{G_0}} \tag{54}$$

$$G_f = \frac{-1}{1 + \frac{1}{G_0} + \frac{1}{G_0}(\frac{s}{\omega_{d1}} + \frac{s}{\omega_{d2}} + \frac{s}{\omega_{d3}}) + \frac{1}{G_0}(\frac{s^2}{\omega_{d1}\omega_{d2}} + \frac{s^2}{\omega_{d2}\omega_{d3}} + \frac{s^2}{\omega_{d3}\omega_{d1}}) + \frac{s^3}{\omega_{d1}\omega_{d2}\omega_{d3}}} \tag{55}$$

Substituting s = jω,

$$G_f = \frac{-1}{\underbrace{1 + \frac{1}{G_0} - \frac{\omega^2}{G_0}\Sigma\frac{1}{\omega_{d1}\omega_{d2}}}_{real} + \underbrace{\frac{j\omega}{G_0}\left(\Sigma\frac{1}{\omega_{d1}} - \frac{\omega^2}{\omega_{d1}\omega_{d2}\omega_{d3}}\right)}_{imaginary}} \tag{56}$$

The imaginary part of the expression vanishes at a frequency given by;

$$\omega^2 = \Sigma\omega_{d1}\omega_{d2} \tag{57}$$

At this frequency, the system transfer function is given by,

$$G_f = \frac{-1}{1 + \frac{1}{G_0} - \frac{\Sigma\omega_{d1}\omega_{d2}}{G_0}\Sigma\frac{1}{\omega_{d1}\omega_{d2}}} \tag{58}$$

$$G_f = \frac{-1}{1 + \frac{1}{G_0} - \frac{(\omega_{d1}\omega_{d2}+\omega_{d2}\omega_{d3}+\omega_{d3}\omega_{d1})}{G_0}\left(\frac{1}{\omega_{d1}\omega_{d2}} + \frac{1}{\omega_{d2}\omega_{d3}} + \frac{1}{\omega_{d3}\omega_{d1}}\right)} \tag{59}$$

The system will be unstable if

$$G_0 + 1 \geq \Sigma\omega_{d1}\omega_{d2}\Sigma\frac{1}{\omega_{d1}\omega_{d2}} \tag{60}$$

To illustrate this point better, let us consider an example where three identical stages are cascaded to obtain the amplifier.

$$G = \frac{-g_0^3}{\left(1 + \frac{s}{\omega_d}\right)^3} \tag{61}$$

$$G_0 = g_0^3 \text{ and } \omega_d = \omega_{d1} = \omega_{d2} = \omega_{d3} \tag{62}$$

$$G_f = \frac{-1}{1 + \frac{1}{g_0^3} - \frac{3\times 3}{g_0^3}} \text{ at } \omega = \sqrt{3}\omega_d \tag{63}$$

$$\tag{64}$$

The system is unstable if the denominator is greater than zero.

$$1 - \frac{8}{g_0^3} > 0 \tag{65}$$

$$g_0 \geq 2 \tag{66}$$

i.e. the system becomes unstable when $g_0 \geq 2$

We could have obtained the same results without computing the overall system gain. It would have sufficed to compute the magnitude of the loop gain when the phase goes to 180°. The stability of the system is dictated by whether the loop gain, when the phase goes to 180°, is greater than 1 or not.

6.1 Frequency Compensation of Higher Order Systems: Order > 2

It is generally understood that the sign of loop gain for a negative feedback system with order > 2 changes from negative to positive at a certain frequency (assuming the system has only poles). If the loop gain at this frequency has a magnitude greater than one, the system becomes unstable (Barkhausen criteria for stability).

It is therefore advisable to keep the order of the negative feedback system always two for the best performance in terms of low steady-state error and high speed (low sensitivity to active device parameter). The best way to compensate a third-order system is to reduce it to second-order by pole-zero compensation technique.

Given that

$$G = \frac{-G_0}{(1 + \frac{s}{\omega_{d1}})(1 + \frac{s}{\omega_{d2}})(1 + \frac{s}{\omega_{d3}})}; \omega_{d3} > \omega_{d2} > \omega_{d1} \tag{67}$$

Introduce zeros such that G is still band-limited

$$G = \frac{-G_0(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})}{(1+\frac{s}{\omega_{d1}})(1+\frac{s}{\omega_{d2}})(1+\frac{s}{\omega_{d3}})} \tag{68}$$

Use one of the zeros to compensate and cancel one of the poles. It is best to cancel ω_{d2} using ω_{z2} and use ω_{z1} to fix the Q of the resultant system at the desirable value.

We will explore other techniques for compensation when we discuss the practical implementations of op-amps.

7 Negative Feedback Using Two-Port Network Theory

Consider the problem of designing a near-ideal amplifier using a practical active device which is non-ideal. Let the immitance matrix of the device be represented by

$$p = \begin{bmatrix} p_{iA} & 0 \\ p_{fA} & p_{0A} \end{bmatrix} \tag{69}$$

It is devoid of any feedback, i.e., it is unilateral but has finite non-zero input immittances. Our goal is to modify this by using a passive network such that the composite matrix becomes

$$p_{composite} = \begin{bmatrix} p_{iA}+p_{if} & p_{rf} \\ p_{fA}+p_{ff} & p_{0A}+p_{0f} \end{bmatrix} \tag{70}$$

where the immittance of the passive network is given by

$$p_{passive} = \begin{bmatrix} p_{if} & p_{rf} \\ p_{ff} & p_{0f} \end{bmatrix} \tag{71}$$

Let us consider equation 70 in more detail. To understand it better, let us invert the matrix.

$$p'_{composite} = \begin{bmatrix} \frac{p_{0A}+p_{0f}}{\Delta p} & \frac{-p_{rf}}{\Delta p} \\ \frac{p_{fA}+p_{ff}}{\Delta p} & \frac{p_{iA}+p_{if}}{\Delta p} \end{bmatrix} \tag{72}$$

where Δp is the determinant of the immitance matrix and is given by

$$\Delta p = (p_{iA} + p_{if})(p_{0A} + p_{0f}) - p_{rf}(p_{fA} + p_{ff}) \tag{73}$$

$$\Delta p = (p_{iA} + p_{if})(p_{0A} + p_{0f}) \left(1 - \underbrace{\frac{p_{rf}(p_{fA} + p_{ff})}{(p_{iA} + p_{if})(p_{0A} + p_{0f})}}_{g_l} \right) \tag{74}$$

g_l is called the loop gain of the feedback arrangement. The system is a positive feedback system if the loop gain is positive, and is a negative feedback system if the loop gain is negative.

Since p_{fA} is normally much greater than p_{ff}, we can rewrite g_l as

$$g_l = \frac{p_{rf} p_{fA}}{(p_{iA} + p_{if})(p_{0A} + p_{0f})} \tag{75}$$

We can now rewrite $p'_{composite}$ as

$$p'_{composite} = \begin{bmatrix} \frac{1}{(p_{iA}+p_{if})(1-g_l)} & \frac{-p_{rf}}{(p_{iA}+p_{if})(p_{0A}+p_{0f})(1-g_l)} \\ \frac{p_{fA}}{(p_{iA}+p_{if})(p_{0A}+p_{0f})(1-g_l)} & \frac{1}{(p_{0A}+p_{0f})(1-g_l)} \end{bmatrix} \tag{76}$$

g_l should be negative for negative feedback systems and made much greater than 1 in magnitude if the system response is to be insensitive to the active device.

$$|g_l| = \left| \frac{p_{rf} p_{fA}}{(p_{iA} + p_{if})(p_{0A} + p_{0f})} \right| \gg 1 \tag{77}$$

This is primarily achieved by

$$|p_{fA}| \gg \left| \frac{(p_{iA} + p_{if})(p_{0A} + p_{0f})}{p_{rf}} \right| \tag{78}$$

Equation 70 now reduces to

$$p'_{compsite} = \begin{bmatrix} \frac{-(p_{0A}+p_{0f})}{p_{rf} p_{fA}} & \frac{1}{p_{fA}} \\ \frac{1}{p_{rf}} & \frac{-(p_{iA}+p_{if})}{p_{rf} p_{fA}} \end{bmatrix} \tag{79}$$

The idealized feedback amplifier will have $p_{fA} = \infty$ and the inverse of the composite matrix reduces to

$$p'_{composite} = \begin{bmatrix} 0 & 0 \\ \frac{1}{p_{rf}} & 0 \end{bmatrix} \tag{80}$$

Let us now suppose that we want to realize an ideal *VCVS*. The ideal VCVS can only be represented by a g-matrix whose elements are given by

$$g = \begin{bmatrix} 0 & 0 \\ g_f & 0 \end{bmatrix} \tag{81}$$

To realize this, we must use its inverse-matrix as the starting point. In this case, the inverse-matrix is the h-matrix. We will term this feedback as *h-feedback*. Since the system is VC (voltage-controlled), the impedance at the input should increase, and hence, the feedback should connect in series at the input. The system is VS (voltage-source) at the output; the output impedance should decrease and hence, the feedback at the output should be in shunt. Figure 23 shows the realization.

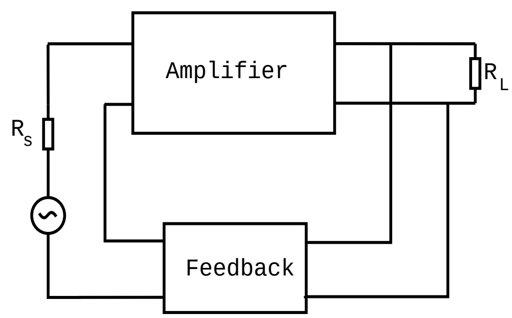

Figure 23: h-feedback for VCVS

Using the same approach as detailed above, we can derive the other basic controlled-sources as well.

CCCS

The ideal CCCS is represented by the h-matrix whose elements are given by

$$g = \begin{bmatrix} 0 & 0 \\ h_f & 0 \end{bmatrix} \tag{82}$$

We must start with its dual, the g-matrix, as the starting point, and hence, the system will be *g-feedback*. Since it is current-controlled at the input, the input impedance should decrease, and hence, the feedback will be in shunt at the input. The output is a current source. The output impedance should therefore increase, and hence, the feedback will be in series at the output. Figure 24 shows the realization.

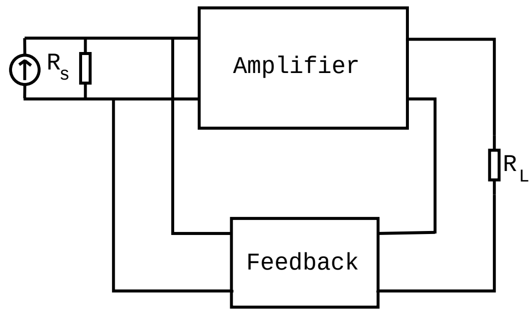

Figure 24: g-feedback for CCCS

<u>VCCS</u>

The ideal VCCS is represented by the Y-matrix whose elements are given by

$$Y = \begin{bmatrix} 0 & 0 \\ Y_f & 0 \end{bmatrix} \tag{83}$$

Since its dual is the Z-matrix, such a system will be *Z-feedback*. The feedback will be series at the input (voltage-controlled) and series at the output (current-source). The realization is shown in figure 25.

<u>CCVS</u>

The ideal CCVS is represented by the Z-matrix whose elements are given by

$$Z = \begin{bmatrix} 0 & 0 \\ Z_f & 0 \end{bmatrix} \tag{84}$$

Since its dual is the Y-matrix, such a system will be *Y-feedback*. The feedback will be shunt at the input (current-controlled) and shunt at the output (voltage-source). The realization is shown in figure 26.

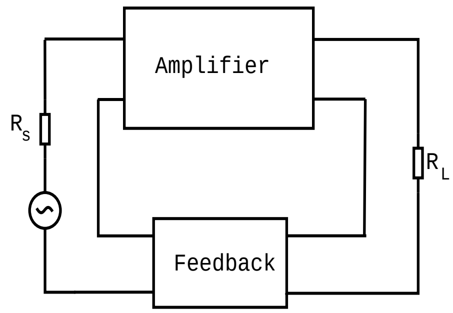

Figure 25: Z-feedback for VCCS

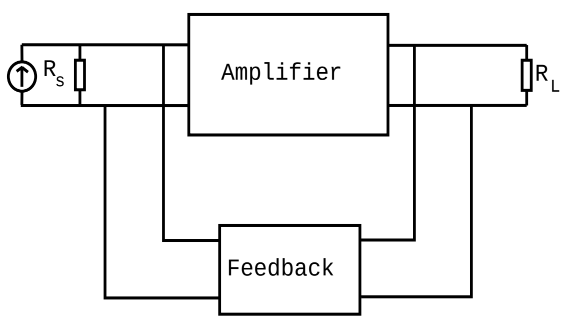

Figure 26: Y-feedback for VCCS

With this we will conclude our basic discussion on feedback theory. In the following chapter, we will design circuits using active devices to realize these wideband feedback amplifier stages.

8 Exercises

[1]. Show that the magnitude variation of G_f with respect to ω is maximally flat for a $Q_0 = \frac{1}{\sqrt{2}}$. How does the step-response look for such a Q_0 ?

[2]. Show that the phase variation of G_f with respect to ω is maximum for $\omega = \omega_0$ and is equal to $\frac{2Q_0}{\omega_0}$.

[3]. What is the value of Q_0 for which a second-order system is critically damped? What is the nature of magnitude response for this value of Q_0 ?

[4]. For a second-order system, show that the magnitude function is maximum at a frequency given by $\omega = \omega_0\sqrt{1 - \frac{1}{2Q_0^2}}$. Also show that the maximum of the magnitude function is equal to $\frac{Q_0}{\sqrt{1-\frac{1}{4Q_0^2}}}$. It can be noted that the peaking is suppressed if the maximum of the magnitude function is set to unity. In such a case, the Q_0 would be equal to $\frac{1}{\sqrt{2}}$

[5]. Show that the maximum overshoot for a step-response of a second-order system (shown in figure 22) is given by, $m_p = e^{\frac{\frac{-\pi}{2Q_0}}{\sqrt{1-\frac{1}{4Q_0^2}}}}$. Further, show that this occurs at a time instant given by $t_p = \frac{\pi}{\omega_0\sqrt{1-\frac{1}{4Q_0^2}}}$ with the ringing occuring at a frequency given by $\omega = \omega_0\sqrt{1 - \frac{1}{4Q_0^2}}$.

[6]. Determine the -3dB bandwidth of a second-order system with a natural frequency of ω_0 and a quality factor of Q_0

[7]. A non-ideal voltage-controlled voltage-source with input resistance of 10K and output resistance of 1K has an open-circuit voltage gain of 1000. If this source is used to design a feedback VCVS using a resistive attenuator of $\frac{1K}{1K+9K}$, determine the input impedance, output impedance and voltage-gain of the feedback amplifier. Evaluate the loop gain. If the loop gain varies with frequency as $\frac{1000}{1+\frac{s}{1000}}$, determine the bandwidth of the feedback amplifier.

[8]. The same non-ideal VCVS is used to realize a transresistor of 1K. Draw the feedback configuration and evaluate the parameters of the feedback amplifier.