Teaching about Attenuators and Compensation: The Time-Domain Approach and the Frequency-Domain Approach

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Prof. Rao, who has taught Analog Design for many years, notes that the Frequency-domain approach is frequently taught in analog classes. In this article, he argues that time should be devoted to the time-domain approach! - Editor

Passive attenuators are important part of electrical and electronic signal processors. Mathematically, they simply cause the variables, voltage or current to get multiplied by a constant value, normally less than one. They are named, in practice, as voltage or current attenuator probes. They are also important in DAC's and ADC's for the purpose of attenuating the reference voltage and getting binary weighted voltage or current needed in these blocks. They once again appear in feedback amplifier design for desensitizing the gain w.r.t. active device parameters. As one has to deal with fast changing signals one has to have no distortion occurring in these blocks. Due to parasitic capacitors occurring at every node in resistive attenuators, distortion is inevitable. Modifying the network to minimize this is called compensation.

Ideal attenuators can be simply constructed from only one type of network element. They could be resistive, capacitive or inductive as shown in Fig.1. The attenuation factor can be computed by the following simple equations, assuming that the output of the attenuator is open-circuit ($i_o = 0$ and hence $i_1 = i_2$) in all cases.

For the resistive attenuator:

$$\frac{V_2}{R_1} = \frac{V_1}{R_1 + R_2} \quad \text{Hence} \quad \frac{V_2}{V_1} = \frac{R_1}{R_1 + R_2} = \frac{1}{10}$$

For the capacitive attenuator:

$$C_1 \frac{dV_4}{dt} = (C_1 \quad C_2)\frac{dV_3}{dt} = \frac{C_1 C_2}{C_1 + C_2}\frac{dV_3}{dt} \quad \text{Hence} \quad \frac{V_4}{V_3} = \frac{C_2}{C_1 + C_2} = \frac{1}{10}$$

Similarly,it can be shown that for the Inductive attenuator,

$$\frac{V_6}{V_5} = \frac{L_1}{L_1 + L_2} = \frac{1}{10}.$$

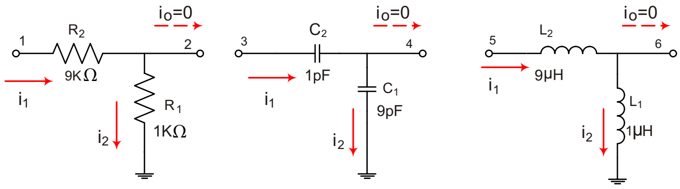


Fig 1 Resistive, Capacitive and Inductive Attenuators

Whenever an attenuator is connected to a device at its output, if a different type of element appears as parasitic, distortion occurs or signal shape changes. Then the attenuator requires to be compensated.

Let us consider the popular oscilloscope 10X probe. It attenuates the voltage by a factor of 10. A typical oscilloscope input resistance is $1\text{M}\Omega$ and it has a parasitic capacitance of 50 pF. In order to design the 10X attenuator, one connects to the input of the scope a $9\text{M}\Omega$ resistor, as shown in Fig.2

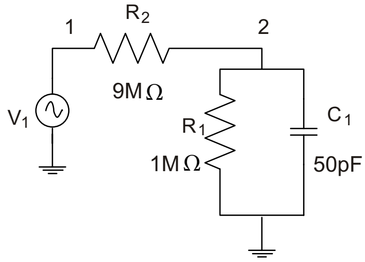


Fig 2 First Version of a 10X Attenuation Probe

Let us excite this probe by a pulse of amplitude 10V and period of 500 ms. We get an exponentially rising and exponentially decaying waveform as shown in Fig.3, reaching the steady state of 1V amplitude. The time constant governing the rise and the fall is

$$\tau = C_1 \frac{R_1 R_2}{R_1 + R_2}$$

It takes roughly five times the time-constant to reach the steady state.

Capacitor current is dependent on the rate of rise of voltage; resistive current depends on the absolute value of voltage. Therefore, whenever the rate of rise is high, to maintain the same attenuation at the same level as when there is no rise, a capacitive attenuator should work in parallel with a resistive attenuator as shown in Fig.4A. The composite compensated attenuator is shown in Fig.4B.

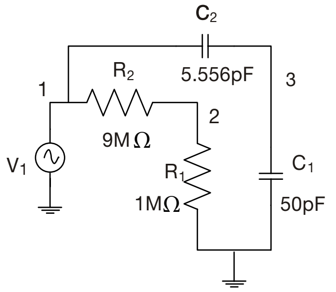


Fig 4A including a Capacitive Attenuator

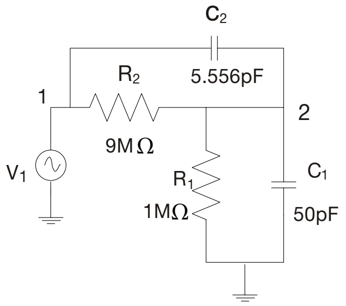


Fig 4B The Composite Attenuator

It is clear that the entire compensation is understood in time domain this way.

This mode of analysis brings out clearly the sensitivities of output to component variations from what is needed for exact compensation.

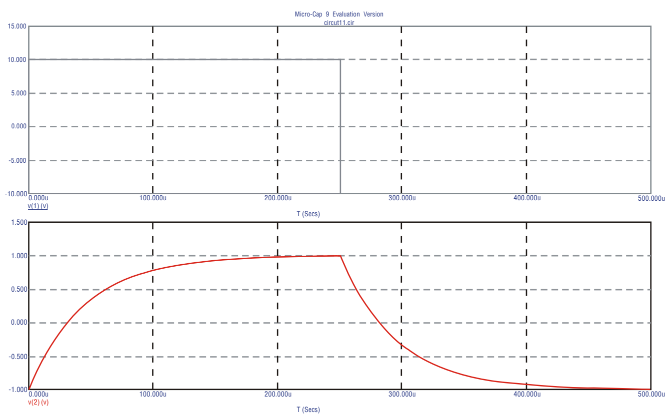
Attenuation for zero rate of rise must be the same as attenuation for infinite rate of rise

i.e. $\frac{R_1}{R_1+R_2} = \frac{C_2}{C_1+C_1}$ and therefore $R_1C_1 = R_2C_2$

Hence $C_2 = \frac{C_1}{9} = 5.556pF$

Fig.5a shows the output of the perfectly compensated attenuator. Fig.5b and Fig. 5c
show the overcompensated (C2=7pF) and under compensated (C2=4pF) situations.

Fig.3



After compensation input,V1 sees $10\ M\Omega$ shunted by a capacitor of value=(55.56/11.11)pF.

Fig.5

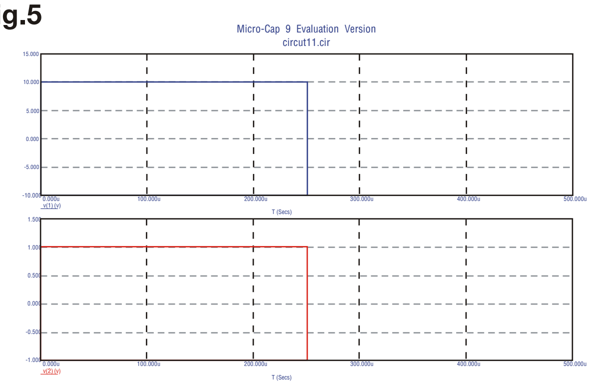


Fig.5a

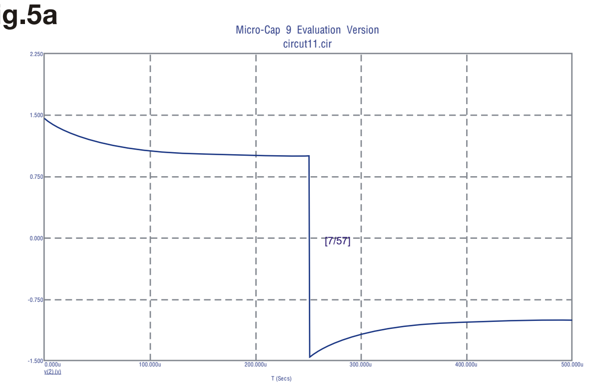


Fig.5b when C2=7pF.

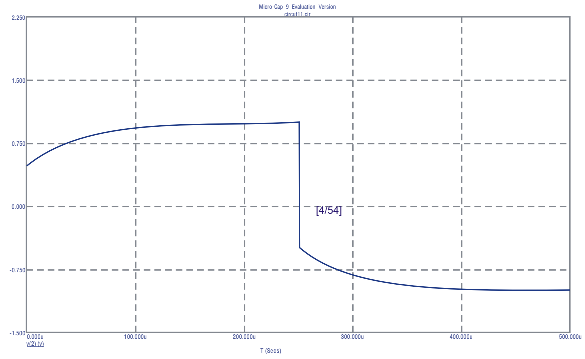


Fig.5c when C2=4pF.

If the time domain analysis is understood, the output waveforms can be easily sketched even for the over compensated (attenuation for infinite rate of rise =7/57) and under compensated cases (attenuation=4/54).

Another way of looking at the problem is using the concept of transfer function and poles and zeros. Let us compare it with the time domain approach which is direct and basic.

The transfer function of the uncompensated network is

$$H(s) = \frac{\frac{R_1}{(R_1+R_2)}}{1+sC_1\frac{R_1R_2}{(R_1+R_2)}}$$

Because of the pole, the magnitude of the transfer function for sinusoidal excitation will decrease with frequency beyond the cut-off frequency $\omega_0 = \frac{1}{C_1\frac{R_1R_2}{(R_1+R_2)}}$

The transfer function of the modified network is

$$H(s) = \frac{R_1}{(R_1 + R_2)} \frac{1 + sC_2R_2}{1 + s(C_1 + C_2)\frac{R_1R_2}{(R_1 + R_2)}}$$

This transfer function has a pole and a zero. By locating the zero at the pole by making

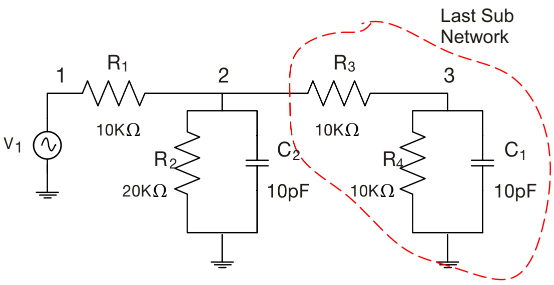
$$R_2C_2 = (C_1 + C_2)\frac{R_1R_2}{(R_1 + R_2)} \quad i.e. \quad \frac{R_2}{(R_1 + R_2)} = \frac{C_2}{(C_1 + C_2)}$$

or, once again: $R_1C_1 = R_2C_2$

and the network has become independent of frequency or wide-band.

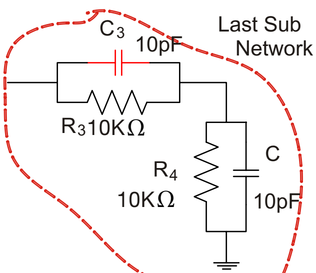
The idea of the above example is to highlight the fact that we must emphasize the time domain approach first and the graduate over to the frequency domain approach as another sophisticated way of looking at systems indirectly.

Now, we will attempt to solve a higher order problem using time domain approach. Let us consider the following resistive R-2R ladder network used in a DAC. Assume existence of node capacitors to ground It is required that it is to be fully compensated.

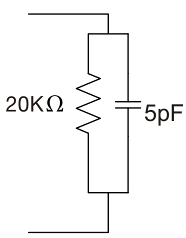


Un-Compensated R-2R Ladder

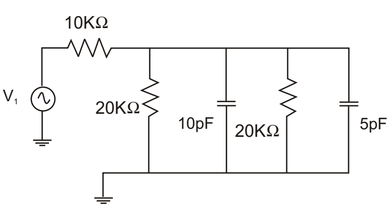
First Method of compensation:



Step1. The last sub-network is first compensated:

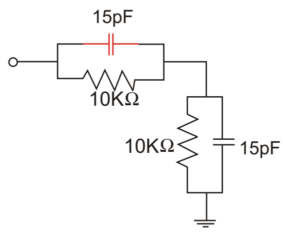


Equivalent to this



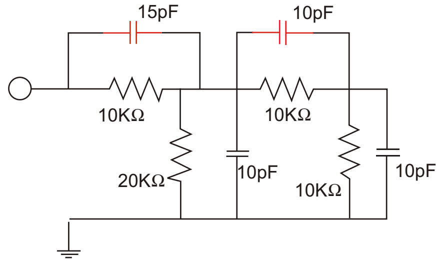
STEP 2: The Composite network now looks like this

This can now be compensated:



The Compensated Network

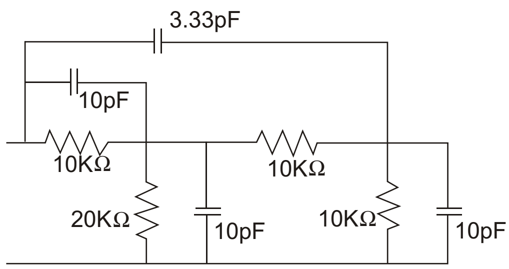
The fully compensated network is:



Try using the same transfer function approach and pole-zero cancellation technique.

Second Method: This method can be termed feed-forward approach. I will give the network and the final values of compensating capacitors. You can readily understand the simplicity of the solution for this. It merely makes the capacitive attenuation at each node same as resistive attenuation.

One can see that second method results in considerable area saving, as the first method requires a total of 25pF as opposed to the second method, which takes only 13.33pF of total capacitor.



Again, try arriving at the second method by pole-zero cancellation technique. Further, sketch the output wave forms at different nodes for a square wave of time period =500 nanoseconds of 1V amplitude when the compensation capacitors deviate from their nominal values by 20%.

Have fun!